# Unit : 1
Graphs and Digraphs

## Q : 1 Short Answer Questions

1. Define **Graph**.
2. What is **Undirected graph**?
3. Define **directed graph**.
4. Define **multiple graph**.
5. What is meaning of **parallel edges**?
6. Define **null graph**.
7. What is **isolated vertex**?
8. Define **pendant vertex**.
9. Define **multiple directed edges**.
10. What is **loop**?
11. Define **simple graph**.
12. Define **multi graph**.
13. What is **pseudo graph**?
14. Define **directed graph**.
15. What is **directed multi graph**?
16. What is **directed multi graph**?
17. Define **simple directed graph**.
18. Define **adjacent nodes**.
19. Define **Incident edges**.
20. What is degree of the vertex in an undirected graph?
21. Define the in-degree of the vertex in a graph with directed edges.
22. What is out-degree of the vertex in a graph with directed edges?
23. Define following terms:
   1) The in-degree of the vertex in a graph with directed edges;
   2) The out-degree of the vertex in a graph with directed edges;
   3) Underlying Undirected graph of a graph with directed edges;
   4) Complete Graph on n vertices.
   5) Regular Graph.
   6) Bipartite Graph.
   7) Complete Bipartite Graph.
   8) Cycle of size n.
   9) Wheel of size n.
   10) n-cube.
   11) Sub-graph of a graph.
   12) Proper sub-graph of a graph.
   13) Spanning sub-graph of a graph.
   14) Vertex deleted sub-graph of a graph.
   15) Edge deleted sub-graph of a graph.
   16) Induced sub-graph of a graph.
   17) Isomorphic Graphs.
   18) Graph Isomorphism.
   19) Invariant property.
   20) Adjacency Matrix.
21) Incidence Matrix.

24. How many numbers of edges are there in $K_n$?
25. How many maximum edges are present in a simple graph with $n$ vertices?

Q: 2 Long Answer Questions.

1. Determine whether the graph shown has directed or undirected edges, whether it has multiple edges, or whether it has one or more loops?

![Graph 1](image1)

2. Determine whether the graph shown has directed or undirected edges, whether it has multiple edges, or whether it has one or more loops?

![Graph 2](image2)

3. Determine whether the graph shown has directed or undirected edges, whether it has multiple edges, or whether it has one or more loops?

![Graph 3](image3)

4. Determine whether the graph shown has directed or undirected edges, whether it has multiple edges, or whether it has one or more loops?

![Graph 4](image4)

5. Determine whether the graph shown has directed or undirected edges, whether it has multiple edges, or whether it has one or more loops?

![Graph 5](image5)
6. Determine whether the graph shown has directed or undirected edges, whether it has multiple edges, or whether it has one or more loops?

7. Does there exist, a simple graph with 5 vertices of the given degrees below? If so draw such a graph.
   (a) 1, 2, 3, 4, 5; (b) 1, 2, 3, 4, 4; (c) 3, 4, 3, 4, 3; (d) 0, 1, 2, 2, 3; (e) 1, 1, 1, 1, 1.

8. Find the number of vertices, number of edges, and degree of each vertex in the following undirected graphs. Identify all isolated and pendant vertices.

9. Find the number of vertices, number of edges, and degree of each vertex in the following undirected graphs and hence, verify the Handshaking Theorem for each one.

10. Find the number of vertices, number of edges, in-degree and out-degree of each vertex in the following directed graphs and hence, verify that the sum of in-degrees, the sum of out degrees of the vertices and the number of edges in the following graphs are equal.
11. Draw the graphs: $K_7$, $K_{(1,8)}$, $K_{(4,4)}$, $C_7$, $W_7$, $Q_4$

12. How many sub-graphs with at least one vertex do (a) $K_2$, (b) $K_3$, (c) $W_3$ have?

13. Determine whether each of these degree sequences represents a simple graph or not? If the graph is possible, draw such a graph.
   (a) 5, 4, 3, 2, 1, 0; (b) 6, 5, 4, 3, 2, 1; (c) 2, 2, 2, 2, 2, 2; (d) 3, 3, 2, 2, 2, 2; (e) 3, 3, 2, 2, 2, 2; (f) 1, 1, 1, 1, 1, 1;
   (g) 5, 3, 3, 3, 3, 3; (h) 5, 5, 4, 3, 2, 1; (i) 3, 3, 3, 3, 2; (j) 4, 4, 3, 2, 1.

14. Represent the following graphs using adjacent matrices:

15. Represent each of these graphs with adjacency matrices:
   (a) $K_4$; (b) $K_{1,4}$; (c) $K_{2,3}$; (d) $C_4$; (e) $W_4$; (f) $Q_3$
16. Draw the graphs with the given adjacency matrix:

(a) \[
\begin{bmatrix}
0 & 1 & 0 \\
1 & 0 & 1 \\
0 & 1 & 0
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
1 & 3 & 2 \\
3 & 0 & 4 \\
2 & 4 & 0
\end{bmatrix}
\]

(c) \[
\begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 1 \\
1 & 1 & 1
\end{bmatrix}
\]

(d) \[
\begin{bmatrix}
1 & 2 & 1 \\
0 & 2 & 0 \\
1 & 1 & 1
\end{bmatrix}
\]

(e) \[
\begin{bmatrix}
0 & 2 & 3 & 0 \\
1 & 2 & 2 & 1 \\
2 & 1 & 1 & 0 \\
1 & 0 & 1 & 0
\end{bmatrix}
\]

(f) \[
\begin{bmatrix}
1 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 \\
1 & 0 & 1 & 1 \\
1 & 1 & 0 & 0
\end{bmatrix}
\]

(g) \[
\begin{bmatrix}
1 & 2 & 0 & 1 \\
2 & 0 & 3 & 0 \\
0 & 3 & 1 & 1 \\
1 & 0 & 1 & 0
\end{bmatrix}
\]

(h) \[
\begin{bmatrix}
0 & 1 & 3 & 0 & 4 \\
1 & 2 & 1 & 3 & 0 \\
3 & 1 & 1 & 0 & 1 \\
0 & 3 & 0 & 0 & 2 \\
4 & 0 & 1 & 2 & 3
\end{bmatrix}
\]

(i) \[
\begin{bmatrix}
0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0
\end{bmatrix}
\]

17. Represent the incidence matrix for the following graphs:

(a) [Graph Image]

(b) [Graph Image]

(c) [Graph Image]

(d) [Graph Image]

(e) [Graph Image]

(f) [Graph Image]

(g) [Graph Image]

18. Determine whether the given graphs are isomorphic or not? Justify by giving appropriate reasons.

(a) [Graph Image]

(b) [Graph Image]

(c) [Graph Image]
19. Determine whether the given pair of directed graphs is isomorphic?
(a)  
(b)  
(c)  
(d)  

20. Draw a call graph for the telephone numbers 555-0011, 555-1221, 555-1333, 555-8888, 555-2222, 555-
0091, and 555-1200 if there were three calls from 555-0011 to 555-8888, two calls from 555-8888 to 555-0011
and two calls from 555-2222 to 555-0091 two calls from 555-1221 to each of the other numbers, and one call
from 555-1333 to each of 555-0011, 555-1221 and 555-1200.

Q: 3 Multiple Choice Questions

1. A graph, in which there is only an edge between a pair of vertices, is called a
   (a) Simple graph.
   (b) Pseudo-graph.
   (c) multi-graph.
   (d) weighted graph.

2. If the degree of a vertex is one, it is called a _________
   (a) Pendant vertex.
   (b) Isolated vertex.
   (c) source.
   (d) sink.

3. A _________ sub-graph of graph G need not contain all its edges.
   (a) Proper
   (b) Spanning
   (c) Induced
   (d) vertex deleted

4. The Handshaking theorem is true for _________ graphs.
   (a) Directed
   (b) Undirected
   (c) Complete
5. The adjacency matrices of two graphs are identical only if the graphs are _______
(a) Simple
(b) Isomorphic
(c) Bipartite
(d) Complete

6. In an adjacency matrix, the $\deg(v_i)$ is equal to the number of ______ in the $i$th row or $i$th column.
(a) 1’s
(b) 0’s
(c) 1’s and 0’s
(d) None of above

7. The adjacency matrix of a simple graph is _______ matrix.
(a) Symmetric
(b) Skew-symmetric
(c) Hermitian.
(d) Skew-Hermitian

8. A graph is a collection of.... ?
(a) Row and columns
(b) Vertices and edges
(c) Equations
(d) None of these

9. The degree of any vertex of graph is .... ?
(a) The number of edges incident with vertex
(b) Number of vertex in a graph
(c) Number of vertices adjacent to that vertex
(d) Number of edges in a graph

10. If for some positive integer $k$, degree of vertex $d(v)=k$ for every vertex $v$ of the graph $G$, then $G$ is called... ?
(a) $K$ graph
(b) $K$-regular graph
(c) Empty graph
(d) All of above

11. A graph with no edges is known as empty graph. Empty graph is also known as... ?
(a) Trivial graph
(b) Regular graph
(c) Bipartite graph
(d) None of these

12. A vertex of a graph is called even or odd depending upon ?
(a) Total number of edges in a graph is even or odd
(b) Total number of vertices in a graph is even or odd
(c) Its degree is even or odd
(d) None of these

13. Which of the following statement is false ?
(a) $G$ is connected and is circuitless
(b) $G$ is connected and has $n$ edges
(c) $G$ is minimally connected graph
14. The given graph is

(d) G is circuitless and has n-1 edges

15. The degrees of {a, b, c, d, e} in the given graph is

(a) 2, 2, 3, 1, 1
(b) 2, 3, 1, 0, 1
(c) 0, 1, 2, 2, 0
(d) 2, 3, 1, 2, 0

16. Which of the following graph is not possible?
(a) Graph with four vertices of degrees 1, 2, 3 and 4.
(b) Graph with four vertices of degrees 1, 2, 3 and 5.
(c) Graph with three vertices of degrees 1, 2 and 3.
(d) Graph with three vertices of degrees 1, 2 and 5.

17. Suppose that a connected planar simple graph has 30 edges. If a plane drawing of this graph has 20 faces, how many vertices does the graph have?
(a) 12
(b) 13
(c) 14
(d) 15

18. The number of distinct simple graphs with up to three nodes is..........
(a) 9
(b) 7
(c) 10
(d) 15

19. Consider the graph G where V(G)={A,B,C,D} and E(G)=[[A,B],[B,C],[C,D]]. The degree of each vertices A,B,C,D respectively in G are.....
(a) 1,2,3,2
(b) 1,3,2,2
20. A graph in which all nodes are of equal degree is known as......
   (a) complete graph
   (b) multi graph
   (c) non regular graph
   (d) regular graph

21. Let $G = (V,E)$ be an undirected graph. Initially all vertices of $G$ are un-marked and all edges are un-labelled. Consider the following algorithm.

   Algorithm $B(G, u)$
   Input: Undirected graph $G$, and vertex $u$ of $G$.

   Mark $u$
   For each edge $(u, v)$ incident on $u$ do {
       if $(u, v)$ is not labelled then {
           if $v$ is not marked then {
               Label $(u, v)$ as discovery
               if $B(G, v) = true$ then return true
           }
       } else return true
   }
   return false

   What does the algorithm do?
   (A) It returns true if none of the edges incident on $u$ is labelled as discovery and it returns false otherwise.
   (B) It returns true if $G$ has at least one cycle and it returns false otherwise.
   (C) It returns true if $G$ is connected and it returns false otherwise.
   (D) It returns true if $u$ has at least one edge labelled discovery and it returns false otherwise.
   (E) It returns true if any vertex $v$ is marked and it returns false otherwise.

22. Let $G = (V,E)$ be an undirected graph. We are interested in selecting a data structure for representing $G$ that allows us to implement the operations areAdjacent($u,v$) and incidentEdges($u$). Let $n$ be the number of vertices and $m$ be the number of edges. Which of the following is true?
   (A) An edge list allows us to perform incidentEdges($u$) in $O(\text{degree}(u))$ time and areAdjacent($u,v$) in $O(1)$ time.
   (B) An adjacency matrix allows us to implement both operations in $O(1)$ time.
   (C) An adjacency list requires $O(1)$ time for implementing incidentEdges($u$) and $O(1)$ time for implementing areAdjacent($u,v$).
   (D) An adjacency matrix can implement areAdjacent($u,v$) in $O(1)$ time and incidentEdges($u,v$) in $O(n)$ time.

Q: 3 True or False.

1. If two graphs are isomorphic, they must have the same number of vertices.
2. If two graphs are isomorphic, they must have the same number of edges.
3. If two graphs are isomorphic, they must have the same degrees for corresponding vertices.
4. If two graphs are isomorphic, they must have the same number of connected components.
5. If two graphs are isomorphic, they must have the same number of loops.
6. If two graphs are isomorphic, they must have the same number of parallel edges.
7. For isomorphism pairs of connected vertices must have the corresponding pair of vertices connected.
8. A complete bipartite graph is one whose vertices can be separated into two disjoint sets where every vertex in one set is connected to every vertex in the other but no vertices within either set are connected.
9. The symbol $K_{n,m}$ is used to denote the complete bipartite graph having $n+1$ vertices in one set and $m$ in the other.
10. A circuit is a closed path and in many books is called a cycle.
11. A simple graph has no loops; each diagonal entry of the adjacency matrix is 0.
12. A row with all 0 entries in an incidence matrix of a graph corresponds to a pendant vertex.
13. A completely bipartite graph need not be a simple graph.
14. All vertex deleted sub-graphs are spanning sub-graphs.
15. The adjacency matrix of a pseudo-graph is a symmetric matrix.
16. There are 18 sub-graphs of $K_{3,3}$ containing at-least one vertex.
17. Two graphs with same vertices and same edges are always isomorphic.
18. The number of vertices of odd degree in an undirected graph is even.
19. In the graph $\text{K}_{3,3}$, the degree of each vertex in the graph is degree 3.
20. Let $G$ be a connected graph. $G$ is an Eulerian graph if and only if the degree of each vertex is odd.

Q: Fill in the blanks.

1. A complete graph, $K_n$, is an ________ -regular graph.
2. A complete graph, $K_n$, has ________ edges.
3. A connected graph has an Eulerian path, which is not a circuit, if it has exactly two vertices of _____ degree.
4. If for some positive integer $k$, degree of vertex $d(v)=k$ for every vertex $v$ of the graph $G$, then $G$ is called ________.
5. A graph with no edges is known as empty graph. Empty graph is also known as ________.
6. A graph $G$ is called a ________ if it is a connected acyclic graph.
7. A row with a single unit entry in an incidence matrix corresponds to a ________ vertex.
8. The adjacency matrices of two ________ graphs are identical.
9. The sum of degrees of all the vertices of an undirected graph is ________ the number of edges of the graph and hence even
10. In any directed graphs, $G=(V,E)$, $\sum_{v\in V} \left\{ \deg^-(v) \right\} = e$
11. The total number of vertices with odd degrees in a graph is always ________.
12. A graph containing ________ is called an Euler graph
13. A graph containing ________ is called a Hamiltonian graph
14. If in an undirected graph, the degree of each vertex is even then the graph contains ________ circuit.
15. If in an undirected graph, the degree of exactly two vertices are odd, then the graph contains ________ path.
16. If a graph has a vertex of degree ________, then it cannot be connected.
17. If a graph has a ________, it must be connected.
18. If $G$ ________ $G'$, then $G$ and $G'$ must have the same number of vertices.
19. The degree sum of any graph is always ________.
20. The total number of edges in the complete graph $K_{12}$ is ________.
Unit – 2
Connectivity, Euler and Hamilton Paths and Circuits

Q : 1 Short Answer Questions.

1. Define following terms:

1) Path in an undirected graph
2) Path in a directed graph
3) Circuit in an undirected graph
4) Circuit in a directed graph
5) Simple path
6) Simple circuit
7) Connected graph
8) Strongly connected graph
9) Weakly connected graph
10) Unilaterally connected graph
11) Euler path
12) Euler circuit
13) Euler graph
14) Hamiltonian path
15) Hamiltonian circuit
16) Hamiltonian graph
17) Strongly connected components
18) Cut vertex
19) Cut edge

2. State difference between the following.
   a) Path and Circuit
   b) Euler Path and Hamiltonian Path
   c) Euler Circuit and Hamiltonian Circuit
   d) Cut vertex and cut edge

3. Give an example of a graph which contains:
   a. an Eulerian circuit that is also a Hamiltonian circuit
   b. an Eulerian circuit, but not a Hamiltonian circuit
   c. a Hamiltonian circuit, but not an Eulerian circuit
   d. neither an Eulerian circuit nor a Hamiltonian circuit
   e. an Eulerian circuit and a Hamiltonian circuit that are distinct

4. State the necessary and sufficient condition for an Euler path in an undirected graph.
5. State the necessary and sufficient condition for an Euler circuit in an undirected graph.

Q : 2 Long Answer Questions.

1. Explain with proper justification, the Konigsberg seven bridges problem with solution.
2. Does each of these lists of vertices form a path in the following graph? Which paths are simple?
   Which are circuits? What are the lengths of those that are paths?
   (a) a,e,b,c,b ; (b) a,e,a,d,b,c,a ; (c) e,b,a,d,b,e ; (d) c,b,d,a,e,c
3. Does each of these lists of vertices form a path in the following graph? Which paths are simple? Which are circuits? What are the lengths of those that are paths?
   (a) a, b, e, c, b;       (b) a, d, a, d, a;       (c) a, d, b, e, a;       (d) a, b, e, c, b, d, a

4. Determine whether the given graphs are connected or not? Also find the connected components for each of the graphs.
   (a) ![Graph A](image1)
   (b) ![Graph B](image2)
   (c) ![Graph C](image3)

5. Determine whether each of these graphs is strongly connected or weakly connected or unilaterally connected?
   ![Graph A](image1)  ![Graph B](image2)  ![Graph C](image3)

6. Determine whether each of these graphs is strongly connected or weakly connected or unilaterally connected?
   ![Graph A](image1)  ![Graph B](image2)  ![Graph C](image3)

7. Find the strongly connected components of each of the given graphs.
   ![Graph A](image1)  ![Graph B](image2)  ![Graph C](image3)
8. Find the strongly connected components of each of the given graphs

9. Find the number of paths of length \( n \) between two different vertices in \( K_4 \) if \( n \) is 
   (a) 2; (b) 3; (c) 4; (d) 5.

10. Find the number of paths of length \( n \) between any two adjacent vertices in \( K_{3,3} \) for the values of \( n \) 
   (a) 2; (b) 3; (c) 4; (d) 5.

11. Find the number of paths between \( c \) and \( d \) in the graph in following figure of length 
   (a) 2; (b) 3; (c) 4; (d) 5; (e) 6; (f) 7.

12. Find the number of paths \( a \) to \( e \) in the following directed graph of length: 
   (a) 2; (b) 3; (c) 4; (d) 5; (e) 6; (f) 7

13. Using simple circuits, check whether the following graphs are isomorphic or not? 
   (a) 
   (b) 
   (c) 
   (d) 

14. Find all the cut vertices and cut edges of the following graphs:
15. Determine whether the given graph has an Euler circuit or an Euler path if it exists, construct it.

16. Can someone cross all the bridges shown in these maps exactly once and return to the starting point?
(a)  
(b)  

17. Determine whether the given graphs have an Euler circuit or an Euler path. If it exists, construct it. Justify your answer by giving appropriate reason.
(a)  
(b)  
(c)  
(d)  
(e)  
(f)
18. Determine whether given graphs have a Hamiltonian Circuit or a Hamiltonian path. If it exists, construct it. Justify your answer by giving appropriate reason.

(a) ![Graph A]
(b) ![Graph B]
(c) ![Graph C]
(d) ![Graph D]

19. Prove that a graph is an Euler graph if and only if it can be decomposed into circuits.

20. Define a Hamiltonian path. Find an example of a non Hamiltonian graph with a Hamiltonian path.

Q: 3 Multiple Choice Questions.

1. If a graph contains a circuit which crosses each edge in the graph exactly once, then such circuit is called
   (a) Hamiltonian circuit
   (b) Euler circuit
   (c) Simple circuit
   (d) None of above

2. If a graph contains a circuit which crosses each vertex in the graph exactly once, then such circuit is called
   (a) Hamiltonian circuit
   (b) Euler circuit
   (c) Simple circuit
   (d) None of above

3. In Königsberg seven bridges problem, the concept of _________ circuit is applied.
   (a) Hamiltonian circuit
   (b) Euler circuit
   (c) Simple circuit
   (d) None of above

4. Length of the walk of a graph is ....
   (a) The number of vertices in walk W
   (b) The number of edges in walk W
   (c) Total number of edges in a graph
   (d) Total number of vertices in a graph

5. Which of the following is not a type of graph?
   (a) Euler
   (b) Hamiltonian
   (c) Tree
   (d) Path
6. A path in graph G, which contains every vertex of G once and only once?
   (a) Euler tour
   (b) Hamiltonian Path
   (c) Euler trail
   (d) Hamiltonian tour

7. The number of circuits that can be created by adding an edge between any two vertices in a tree is?
   (a) Two
   (b) Exactly one
   (c) At least two
   (d) None

8. The Hamiltonian circuit for the following graph is

![Graph with vertices and edges]

(a) abcdefgh
(b) abefgha
(c) abcdefgha
(d) None

9. Euler formula for graphs is
   (a) $f = e - v$
   (b) $f = e + v + 2$
   (c) $f = e - v - 2$
   (d) $f = e - v + 2$

10. The graph given below
    ![Graph with vertices and edges]
    (a) Has Euler circuit
    (b) Has Hamiltonian circuit
    (c) Does not have Hamiltonian circuit
    (d) None of the above

11. If a graph has any vertex of degree 3 then
    (a) It must have Euler circuit
    (b) It must have Hamiltonian circuit
    (c) It does not have Euler circuit
    (d) None of the above

12. The length of Hamiltonian Path in a connected graph of n vertices is
    (a) $n - 1$
    (b) $n$
    (c) $n + 1$
    (d) $n/2$
13. An undirected graph possesses an eulerian circuit if and only if it is connected and its vertices are
   (a) all of even degree
   (b) all of odd degree
   (c) of any degree
   (d) even in number
14. Suppose I have a connected graph with 7 vertices, of degrees 1, 2, 4, 4, 6, 8, and 9. Does the graph have an Euler path or Euler circuit?
   (a) It has an Euler circuit
   (b) It has an Euler path, but no Euler circuit
   (c) It has neither an Euler path nor an Euler circuit
   (d) Not enough information to tell
15. What if instead the graph had vertices of degrees 1, 2, 2, 3, 3, and 3?
   (a) It has an Euler circuit
   (b) It has an Euler path, but no Euler circuit
   (c) It has neither an Euler path nor an Euler circuit
   (d) Not enough information to tell
16. What if instead the graph had vertices of degrees 2, 4, 6, 8, 10, and 20?
   (a) It has an Euler circuit
   (b) It has an Euler path, but no Euler circuit
   (c) It has neither an Euler path nor an Euler circuit
   (d) Not enough information to tell
17. Let’s change the question slightly. Suppose a connected graph has 4 vertices of degrees 3, 3, 3, and 3. Does it have a Hamilton path or circuit?
   (a) It has a Hamilton circuit
   (b) It has a Hamilton path, but no Hamilton Circuit
   (c) It has neither a Hamilton path nor a Hamilton circuit
   (d) Not enough information to tell
18. Which is a circuit that traverses each edge of the graph exactly once?
   (a) Euler circuit
   (b) Hamilton circuit
   (c) Minimum Spanning Tree
   (d) any circuit
19. Which is a circuit that traverses vertex of the graph exactly once?
   (a) Euler circuit
   (b) Hamilton circuit
   (c) Minimum Spanning Tree
   (d) any circuit
20. If a graph is connected and __________, the graph will have an euler circuit.
   (a) the graph has an even number of vertices
   (b) the graph has an even number of edges
   (c) the graph has all vertices of even values
   (d) the graph has only two odd vertices

Q: 4 True or False.

1. The total degree of a graph determines if a graph has a path or not.
2. A graph can have exactly 5 even degree vertices.
3. Euler’s theorem shows how to find a Euler circuit in a graph.
4. A disconnected graph may have isolated vertices.
5. An exhaustive route can retrace edges.
6. The total degree of a connected graph must be even.
7. A loop is a path of length one.
8. An Euler circuit is also an Euler path.
9. If G is isomorphic to G' and G has an Euler circuit, then G' also has one.
10. If G and G' both have Euler circuits, then G is isomorphic to G'.
11. If exactly two vertices of G have odd degree, then G has an Euler circuit.
12. A graph cannot have an Euler circuit if it is not connected.
13. A graph with exactly two vertices of odd degree has a minimum Eulerization with just one additional edge.
14. A graph with exactly two vertices of odd degree, and that are also adjacent, has a minimum Eulerization with just one additional edge.
15. A graph has only one minimum Eulerization.
16. In an Eulerized graph, the number of times a vertex (other than the base vertex) appears in any Euler circuit must equal one-half of its degree.
17. Every connected graph must have a Hamilton circuit.
18. If G and G' are isomorphic to each other and G has a Hamilton circuit, then so does G'.
19. If G and G' both do not have a Hamilton circuit, then they must be isomorphic to each other.
20. The complete graph K6 contains 60 Hamilton circuits.

Q: 5 Fill in the blanks.
1. ______ term refers to the process of retracing some streets on a parade route in order to have a parade go down all the streets in a neighborhood.
2. Two vertices connected by a single edge called ______.
3. The picture representation of a city neighborhood called __________.
4. ________ term refers to process of finding an Euler circuit in a graph that has an Euler Circuit.
5. _______ concept states that there can only be an even number of odd vertices on a graph?
6. The total number of Hamilton circuits in K8 is ____________.
7. The degree sum of a graph with 150 edges is ____________.
8. The number of edges contained in a graph having 500 vertices each of degree 8 is ________.
9. A ________ is one current path that connects two nodes.
10. A node is the junction of two or more ________.
11. For a three-loop circuit, three __________ equations are required.
12. The node voltage method of circuit analysis is based on finding the voltages at each node in the circuit using Kirchhoff’s ______.law.
13. The first step in using the node voltage method of circuit analysis is to determine the ______ of nodes.
14. Two elements, branches, or networks are in parallel if they have two _______ in common.
15. The length of the longest simple circuit in $K_5$ is ________.
16. The length of the longest simple circuit in $W_{10}$ is ________.
17. $W_n$ has _______ edges and _______ vertices.
18. List all positive integers $n$ such that $K_n$ has an Euler circuit. ________.
19. List all positive integers $n$ such that $Q_n$ has an Euler circuit. ________.
20. List all positive integers $n$ such that $W_n$ has an Euler circuit. ________.
Unit – 3
Shortest-Path Problems and Algorithms

Q : 1 Long Answer Questions.

1. Give the step by step procedure of Dijkstra’s algorithm to find the shortest path between any two vertices.
2. Give the step by step procedure of Floyd’s algorithm to find the shortest path between any two vertices.
3. Give the step by step procedure of Chinese postman algorithm to find the optimal route.
4. Find the shortest path from the vertices a to z in the following graph model:

![Graph Model 1](image1)

5. Find the shortest path for the graph model given below:

![Graph Model 2](image2)

6. Find the shortest path from the vertices a to z in the following graph model:

![Graph Model 3](image3)

7. Find the shortest path from the vertices a to z in the following graph model:

![Graph Model 4](image4)
8. Solve the Travelling Salesman problem for the graph by finding the total weight of all Hamiltonian circuits and determining a circuit with minimum total weight.
(a) 
(b) 
(c) 
(d) 
(e) 

9. Find the optimal route of the following graphs of Chinese Postman Problem:

10. Find the optimal route of the following graphs of Chinese Postman Problem:

11. Find the shortest path between each pair of vertices by using Floyd’s Algorithm:
12. Find the shortest path between each pair of vertices by using Floyd’s Algorithm:

13. Find the shortest path between each pair of vertices by using Floyd’s Algorithm:

14. Find the shortest path between each pair of vertices by using Floyd’s Algorithm:

15. Find shortest path form Vj to v8 using Dijkstra algorithm in the following graph.
16. Find the shortest distance from A to J on the network below.
17. Execute the dijkstra algorithm on the following graph, with single-source $s$.

18. Find the shortest distance from A to H on the network below.

19. The diagram below shows roads connecting towns near to Rochdale. The numbers on each arc represent the time, in minutes, required to travel along each road. Peter is delivering books from his base at Rochdale to Stockport. Use Dijkstra’s algorithm to find the minimum time for Peter’s journey.
20. The diagram below shows roads connecting villages near to Royton. The numbers on each arc represent the distance, in miles, along each road. Leon lives in Royton and works in Ashton. Use Dijkstra’s algorithm to find the minimum distance for Leon’s journey to work.

![Diagram showing roads connecting villages near Royton.](image)

21. Find a solution to the Chinese Postman Problem in this graph, given that every edge has equal weight.

![Graph](image)

**Q: 2 Multiple Choice Questions.**

1. In Travelling Salesman Problem, the concept of ________ circuit is applied.
   (a) Hamiltonian circuit (b) Euler circuit (c) Simple circuit (d) None of above

2. In Chinese Postman Problem, the concept of ________ circuit is applied.
   (a) Hamiltonian circuit (b) Euler circuit (c) Simple circuit (d) None of above

3. Which algorithm solves all pair shortest path problem
   (a) Dijkstra’s algorithm (b) Floyd’s algorithm (c) Prim’s algorithm (d) Worshal’s algorithm

4. When compared to the travelling salesman problem with $n$ nodes, vehicle routing problems with $n$ nodes are:
   (a) Computationally easier to solve, since each tour in the VRP has fewer nodes than the TSP.
   (b) Computationally the same, since there is an equal number of customer nodes.
   (c) Computationally harder, since you must determine both who is on which tour and how to build the tours.
   (d) Computationally the same or harder, depending on the tour constraints.

5. A feasible tour for a travelling salesman problem with 45 cities (including the “home” city) has
   (a) 27 arcs (b) 44 arcs (c) 45 arcs (d) 990 arcs

6. To implement Dijkstra’s shortest path algorithm on unweighted graphs so that it runs in linear time, the data structure to be used is:
   (a) Queue (b) stack (c) Heap (d) B-Tree

7. Consider the following graph. Assume that we use Dijkstra’s algorithm to find shortest paths from vertex $s$ to the other vertices in the graph. In which order are the final distance labels $u.d$ computed? (Or in other words, in which order are the shortest paths computed?)
8. Consider the directed graph shown in the figure below. There are multiple shortest paths between vertices S and T. Which one will be reported by Dijkstra’s shortest path algorithm? Assume that, in any iteration, the shortest path to a vertex v is updated only when a strictly shorter path to v is discovered.

(A) SDT
(B) SBDT
(C) SACDT
(D) SACET

9. Dijkstra’s single source shortest path algorithm when run from vertex a in the below graph, computes the correct shortest path distance to:

(A) only vertex a
(B) only vertices a, e, f, g, h
(C) only vertices a, b, c, d
(D) all the vertices

10. In an unweighted, undirected connected graph, the shortest path from a node S to every other node is computed most efficiently, in terms of time complexity by

(A) Dijkstra’s algorithm starting from S.
(B) Warshall’s algorithm
(C) Performing a DFS starting from S.
11. Suppose we run Dijkstra’s single source shortest-path algorithm on the following edge weighted directed graph with vertex P as the source. In what order do the nodes get included into the set of vertices for which the shortest path distances are finalized?

(A) P, Q, R, S, T, U
(B) P, Q, R, U, S, T
(C) P, Q, R, U, T, S
(D) P, Q, T, R, U, S

12. Dijkstra’s algorithm is used to _______.
   (A) Create LSAs
   (B) Flood an internet with information
   (C) Create a link state database
   (D) Calculate the routing tables

UNIT – 4
Trees
Q : 1 Short Answer Questions.
1. Define following.
   1) Tree
   2) Forest
   3) Rooted Tree
   4) Sub-tree
   5) Parent of vertex v in a rooted tree
   6) Child of vertex v in a rooted tree
   7) Sibling of vertex v in a rooted tree
   8) Ancestor of vertex v in a rooted tree
   9) Descendant of vertex v in a rooted tree
   10) Internal vertex
   11) Leaf
   12) Level of a vertex
   13) Height of a vertex
   14) m-ary tree
   15) Full m-ary tree
   16) Complete m-ary tree
   17) ALV Tree
   18) B Tree
   19) Tries
   20) Red-Black Trees
21) Splay Trees

2. Give an example of a full 3-ary tree.
3. How many leaves and internal vertices does a full binary tree with 25 total vertices have?
4. What are the maximum and minimum heights of a binary tree with 25 vertices have?
5. In each of the following cases, state whether or not such a tree is possible.
   (a) A binary tree with 35 leaves and height 100.
   (b) A full binary tree with 21 leaves and height 21.
   (c) A binary tree with 33 leaves and height 5.
   (d) A rooted tree of height 5 where every internal vertex has 3 children and there are 365 vertices.
6. Given two vertices in a tree, how many distinct simple paths can we find between the two vertices?

Q : 2 long answer questions.

1. Which of the given graphs are trees?

2. Which of the given graphs are trees?

3. Answer the following questions about the rooted trees given below:

   (a) Which vertex is the root?
   (b) Which vertices are internal?
5. Answer the following questions about the rooted trees given below:

(a) Which vertices are descendants of b?
(b) Is the rooted tree a full m-ary tree for some positive integer m?
(c) What is the level of each vertex of the rooted tree?
(d) Draw a sub-tree of the tree in the given tree figures that is rooted at (a) a; (b) c; (c) b

6. Construct a complete binary tree of height 4 and a complete 3-ary tree of height 3.

7. How many nonisomorphic unrooted trees are there with three vertices?
8. How many edges does a tree with 10,000 vertices have?
9. How many vertices does a fully binary tree with 1000 internal vertices have?
10. How many leaves does a fully 3-ary tree with 100 vertices have?
11. How many edges does a full binary tree with 1000 internal vertices have?
12. Draw the subtree of the tree that is rooted at “a, c, e”

13. Draw the subtree of the tree that is rooted at “a, c, e”

14.  
   a) How many non isomorphic unrooted trees are there with three vertices?
   b) How many non isomorphic rooted trees are there with three vertices (using isomorphism for directed graphs?)
15. What is the level of each vertex of the rooted tree

16. What is the level of each vertex of the rooted tree

Q: 3 Multiple Choice Questions.

1. A tree is
   A) any graph that is connected and every edge is a bridge.
   B) any graph that has no circuits.
   C) any graph with one component.
   D) any graph that has no bridges.

2. Graph 1 is connected and has no circuits. Graph 2 is such that for any pair of vertices in the graph there is one and only one path joining them.
   A) Graph 1 cannot be a tree; Graph 2 cannot be a tree.
   B) Graph 1 must be a tree; Graph 2 may or may not be a tree.
   C) Graph 1 must be a tree; Graph 2 must be a tree.
   D) Graph 1 must be a tree; Graph 2 cannot be a tree.

3. Suppose T is a tree with 21 vertices. Then
   A) T has one bridge.
   B) T has no bridges.
   C) T can have any number of bridges.
   D) T has 20 bridges.

4. In a tree between every pair of vertices there is?
   A) Exactly one path
   B) A self loop
   C) Two circuits
   D) n number of paths

5. If a tree has 8 vertices then it has
   A) 6 edges
   B) 7 edges
   C) 9 edges
   D) None of the above

6. A graph is tree if and only if
   A) Is planar
   B) Contains a circuit
7. Suppose that A is an array storing n identical integer values. Which of the following sorting algorithms has the smallest running time when given as input this array A?
   A) Insertion Sort
   B) Selection Sort
   C) Ordered-dictionary sort implemented using an AVL tree.
   D) Ordered-dictionary sort implemented using a (2,4)-tree.

8. How many different (2, 4)-trees containing the keys 1, 2, 3, 4, and 5 exist (each key must appears once in each one of these (2, 4)-trees)?
   (A) 2   (B) 3   (C) 4   (D) 5

9. For which of the following does there exist a graph G = (V, E, φ) satisfying the specified conditions?
   A) A tree with 9 vertices and the sum of the degrees of all the vertices is 18.
   B) A graph with 5 components 12 vertices and 7 edges.
   C) A graph with 5 components 30 vertices and 24 edges.
   D) A graph with 9 vertices, 9 edges, and no cycles.

10. For which of the following does there exist a simple graph G = (V, E) satisfying the specified conditions?
   (A) It has 3 components 20 vertices and 16 edges.
   (B) It has 6 vertices, 11 edges, and more than one component.
   (C) It is connected and has 10 edges 5 vertices and fewer than 6 cycles.
   (D) It has 7 vertices, 10 edges, and more than two components.

11. For which of the following does there exist a tree satisfying the specified constraints?
    A) A binary tree with 65 leaves and height 6.
    B) A binary tree with 33 leaves and height 5.
    C) A full binary tree with height 5 and 64 total vertices.
    D) A rooted tree of height 3, every vertex has at most 3 children. There are 40 total vertices.

12. For which of the following does there exist a tree satisfying the specified constraints?
    A) A full binary tree with 31 leaves, each leaf of height 5.
    B) A rooted tree of height 3 where every vertex has at most 3 children and there are 41 total vertices.
    C) A full binary tree with 11 vertices and height 6.
    D) A binary tree with 2 leaves and height 100.

13. The number of simple digraphs with |V| = 3 is
    (a) 2^6
    (b) 2^5
    (c) 2^7
    (d) 2^6

14. The number of simple digraphs with |V| = 3 and exactly 3 edges is
    (a) 92
    (b) 88
    (c) 80
    (d) 84

15. The number of oriented simple graphs with |V| = 3 is
    (a) 27
    (b) 24
    (c) 21
    (d) 18
16. The number of oriented simple graphs with $|V| = 4$ and 2 edges is
   (a) 40
   (b) 50
   (c) 60
   (d) 70

17. Which data structure allows deleting data elements from front and inserting at rear?
   (a) Stacks
   (b) Queues
   (c) Deques
   (d) Binary search tree

18. To represent hierarchical relationship between elements, which data structure is suitable?
   (a) Deque
   (b) Priority
   (c) Tree
   (d) All of above

19. A binary tree whose every node has either zero or two children is called
   (a) Complete binary tree
   (b) Binary search tree
   (c) Extended binary tree
   (d) None of above

20. A binary tree can easily be converted into q 2-tree
   (a) by replacing each empty sub tree by a new internal node
   (b) by inserting an internal nodes for non-empty node
   (c) by inserting an external nodes for non-empty node
   (d) by replacing each empty sub tree by a new external node

UNIT - 5
Trees Traversal

Q : 1 Short Answer Questions.

1. Define following.
   1) Binary Tree
   2) Ordered Tree
   3) Balanced Tree
   4) Binary search Tree
   5) Decision Tree
   6) Prefix Code
   7) Minmax strategy
   8) Value of a vertex in a game tree
   9) Tree Traversal
   10) Preorder Traversal
   11) Inorder Traversal
   12) Postorder Traversal
   13) Infix notation
   14) Prefix(or polish) notation
   15) Postfix (or reverse Polish) notation
   16) Define Universal address system.

2. Give an example of a graph that satisfies the specified condition or show that no such graph exists.
3. A tree with six vertices and six edges.
4. A tree with three or more vertices, two vertices of degree one and all the other vertices with degree three or more.
5. A disconnected graph with 10 vertices and 8 edges.
6. A disconnected graph with 12 vertices and 11 edges and no cycle.
7. A tree with 6 vertices and the sum of the degrees of all vertices is 12.
8. A connected graph with 6 edges, 4 vertices, and exactly 2 cycles.
9. A graph with 6 vertices, 6 edges and no cycles.

Q : 2 Long Answer Questions.

2. Give the step by step procedure of Inorder Traversal of Trees and its algorithm.
4. Represent the expressions \((x+2)^3 \cdot (y - (3 + x)) - 5\) using a binary tree
   Write this expression in
   Prefix notation
   Postfix notation
   Infix notation
5. Represent the expression \((x + xy) + (x/y)\) and \(x + ((xy + x)/y)\) using binary tree
   Write this expression in
   Prefix notation
   Postfix notation
   Infix notation
6. In which order are the vertices of the ordered rooted tree in below tree visited using an inorder traversal?
7. Build a BST from the following lists:
   a. 6, 4, 7
   b. 6, 4, 7, 2, 5, 9
8. Build a BST from these inputs: 10, 20, 30, 40, 5, 8, 50, 60, 70, 15, 80
9. In which order are the vertices of the ordered rooted tree visited using an inorder traversal?

10. In which order are the vertices of the ordered rooted tree visited using an inorder traversal?

11. In which order are the vertices of the ordered rooted tree visited using an inorder traversal?
12. In which order are the vertices of the ordered rooted tree visited using a postorder traversal?

13. In which order are the vertices of the ordered rooted tree visited using a postorder traversal?

14. In which order are the vertices of the ordered rooted tree visited using a postorder traversal?

15. Show that preorder traversals of the two ordered rooted trees displayed below produce the same list of vertices.
16. Show that postorder traversals of these two ordered rooted trees produce the same list of vertices.

![Ordered Rooted Trees Diagram]

17. What is the value of each of these prefix expressions?
   a) − * 2 / 8 4 3
   b) ↑ − * 3 3 * 4 2 5
   c) + − ↑ 3 2 ↑ 2 3 / 6 − 4 2
   d) * + 3 + 3 ↑ 3 + 3 3 3

18. What is the value of each of these postfix expressions?
   a) 5 2 1 − − 3 1 4 ++ *
   b) 9 3 / 5 + 7 2 − *
   c) 3 2 * 2 ↑ 5 3 − 8 4 / * −

19. Draw the ordered rooted tree corresponding to each of these arithmetic expressions written in prefix notation. Then write each expression using infix notation.
   a) + * + − 5 3 2 1 4
   b) ↑ + 2 3 − 5 1
   c) * / 9 3 * * 2 4 − 7 6

20. Represent \((A \cap B) - (A \cup (B - A))\) using an ordered rooted tree. Write this expression in
   a) prefix notation.
   b) postfix notation.
   c) infix notation.

21. Suppose that the vertex with the largest address in an ordered rooted tree \(T\) has address 2.3.4.3.1. Is it possible to determine the number of vertices in \(T\)?

22. Construct the ordered rooted tree whose preorder traversal is a, b, f, c, g, h, i, d, e, j, k, l, where a has four children, c has three children, j has two children, b and e have one child each, and all other vertices are leaves.

Q: 3 Multiple Choice Questions

1. If \(T\) is a full binary tree and has 5 internal vertices then the total vertices of \(T\) are
   (a) 11
   (b) 12
   (c) 13
   (d) None of the these

2. We are given a set of \(n\) distinct elements and an unlabeled binary tree with \(n\) nodes. In how many ways can we populate the tree with the given set so that it becomes a binary search tree?
   (a) 0
   (b) 1
   (c) \(n\)
   (d) \((1/(n+1)).2nCn\)

3. Consider the following algorithm:
   Algorithm T(r)
   Input: Root \(r\) of a proper binary tree.
   if \(r\) is a leaf then return 0
   else {
     \(p \ T(\text{left child of } r)\)
q  T(right child of r)
    if p > q then return p + 1
    else return q + 1
}  

What does the algorithm compute?
(A) The number of nodes in the tree.
(B) The number of internal nodes in the tree.
(C) The number of nodes in the largest subtree of r.
(D) The height of the tree.

4. Define an RP-tree by the parent-child adjacency lists as follows:
   (i) Root B: J, H, K; (ii) H: P, Q, R; (iii) Q: S, T; (iv) K: L, M, N. The postorder vertex sequence of this tree is
   (A) J, P, S, T, Q, R, H, L, M, N, K, B.
   (B) P, S, T, J, Q, R, H, L, M, N, K, B.
   (C) P, S, T, Q, R, H, L, M, N, K, J, B.
   (D) P, S, T, Q, R, J, H, L, M, N, K, B.

5. Define an RP-tree by the parent-child adjacency lists as follows:
   (i) Root B: J, H, K; (ii) J: P, Q, R; (iii) Q: S, T; (iv) K: L, M, N. The preorder vertex sequence of this tree is
   (A) B, J, H, K, P, Q, R, L, M, N, S, T.
   (B) B, J, P, Q, S, T, R, H, K, L, M, N.
   (C) B, J, P, Q, S, T, R, H, L, M, N, K.
   (D) B, J, Q, P, S, T, R, H, L, M, N, K.

6. The postfix form of A*B+C/D is
   (A) *AB/CD+
   (B) AB*CD/+  
   (C) A*BC+/D
   (D) ABCD+/*

7. One can make an exact replica of a Binary Search Tree by traversing it in
   (A) Inorder
   (B) Preorder
   (C) Postorder
   (D) Any order

8. The post order traversal of a binary tree is DEBFCA. Find out the preorder traversal.
   (A) ABFCDE
   (B) ADBFEC
   (C) ABDECF
   (D) ABDCFE

9. Preorder is nothing but
   (A) Depth first order
   (B) Breadth first order
   (C) Topological order
   (D) Linear order

10. What is the postfix form of the following prefix expression -A/B+C$DE
     (A) ABCDE$*/-
     (B) A-BCDE$*/-
     (C) ABC$ED*/-
     (D) A-BCDE$*/

11. One can convert a binary tree into its mirror image by traversing it in
(A) Inorder  
(B) Preorder  
(C) Postorder  
(D) Any order
UNIT – 6
Spanning Trees and Minimum Spanning Trees

Q : 1 Short Answer Questions.

1. Define following.
   - Spanning Tree
   - Minimum Spanning Tree
   - State difference between following:
     - Spanning Tree and minimum spanning tree
     - Binary tree and rooted tree
     - Preorder transversal and postorder transversal
     - Decision tree and Balanced tree

2. Draw all the spanning trees of this graph:

3. What is the purpose of Spanning Tree Protocol in a switched LAN?

4. What is Spanning tree?

5. What is non-designated port?

6. What is the use of Spanning Tree Protocol (STP)?

7. Justify the statement" Every connected graph has a spanning tree."

8. Assuming that G₀ has a minimum spanning T₀, is it true that the number of edges in the T₀ no greater than T?
   Answer yes or no and your answer explain in one sentence.

9. Assuming that G’ has a minimum spanning tree T’, by how many edges can T’ differ from T.

10. How many spanning tree does the following graph have?

11. How many spanning tree does the following graph have?
12. How many spanning trees does the following graph have?

13. Is the minimum weight edge in a graph always in any minimum spanning tree for a graph? Is it always in any single source shortest path tree for the graph? Justify your answers.
15. State whether your minimum spanning tree is unique.

**Q : 2 long answer questions.**

1. Draw all the spanning trees of $k_3$.
2. Give the step by step procedure of Prim’s algorithm to find the minimum spanning tree.
3. Give the step by step procedure of Kruskal’s algorithm to find the minimum spanning tree.
4. For each of the following graphs:

   (a) Find all spanning trees.
   (b) Find all spanning trees up to isomorphism.
   (c) Find all depth-first spanning trees rooted at A.
   (d) Find all depth-first spanning trees rooted at B.

5. For each of the following graphs:

   (a) Find all minimum spanning trees.
   (b) Find all minimum spanning trees up to isomorphism.
   (c) Among all depth-first spanning trees rooted at A, find those of minimum weight.
   (d) Among all depth-first spanning trees rooted at B, find those of minimum weight.

6. In the following graph, the edges are weighted either 1, 2, 3, or 4.

Referring to discussion following of Kruskal’s algorithm:
(a) Find a minimum spanning tree using Prim’s algorithm
(b) Find a minimum spanning tree using Kruskal’s algorithm.
(c) Find a depth-first spanning tree rooted at K.

7. Let \( T = (V,E) \) be a tree and let \( d(v) \) be the degree of a vertex
   (a) Prove that \( \sum_{v \in V} (2 - d(v)) = 2 \).
   (b) Prove that, if \( T \) has a vertex of degree \( m \), then it has at least \( m \) vertices of degree 1.
   (c) Give an example for all \( m \) of a tree with a vertex of degree \( m \) and only \( m \) leaves.

8. Does every connected graph have a spanning tree? Either give a proof or a counter-example.
9. Give an algorithm that determines whether a graph has a spanning tree, and finds such a tree if it exists, that takes time bounded above by a polynomial in \( v \) and \( e \), where \( v \) is the number of vertices, and \( e \) is the number of edges.
10. Find all spanning trees (list their edge sets) of the graph

11. Show that a finite graph is connected if and only if it has a spanning tree.
12. Find spanning tree of maximal(minimal) weights for graphs from figures using algorithms of Kruskal and Prim.

13. Prove that every minimal spanning tree of a connected graph may be constructed by Kruskal’s algorithm.
14. Use Kruskal’s algorithm to find all least weight spanning trees for this weighted graph.

15. Find a minimum weight spanning tree in each of the following weighted graph
16. Find a minimum weight spanning tree in each of the following weighted graph:

17. Draw all the spanning trees of \( K_4 \).

18. Find the number of spanning trees in

19. a. Use Kruskal’s algorithm to find a minimal spanning tree for the graph below.
b. Use Prim’s algorithm (as shown in class) to find a minimal spanning tree different from the one in part

Q: 3 Multiple Choice Questions.

1. How many spanning trees does the following graph have?

   A) 1  
   B) 2  
   C) 3  
   D) 4

The question(s) that follow refer to the problem of finding the minimum spanning tree for the weighted graph shown below.
3. In Figure, using Kruskal's algorithm, which edge should we choose first?
   A) AB
   B) EG
   C) BE
   D) AG

4. In Figure, using Kruskal's algorithm, which edge should we choose third?
   A) EF
   B) AG
   C) BG
   D) EG

5. In Figure, using Kruskal's algorithm, which edge should we choose last?
   A) AB
   B) AC
   C) CD
   D) BC

6. In figure, which of the following edges of the given graph are not part of the minimum spanning tree?
   A) AC
   B) EF
   C) AG
   D) BG

7. In Figure, the total weight of the minimum spanning tree is
   A) 36.
   B) 42.
   C) 55.
   D) 95.

8. Which of the following statements is true about Kruskal's algorithm.
   A) It is an inefficient algorithm, and it never gives the minimum spanning tree.
   B) It is an efficient algorithm, and it always gives the minimum spanning tree.
   C) It is an efficient algorithm, but it doesn't always give the minimum spanning tree.
   D) It is an inefficient algorithm, but it always gives the minimum spanning tree.

For the following question(s), refer to the digraph below.
9. In Figure, which of the following is not a path from vertex E to vertex B in the digraph?
   A) E, B, C, B
   B) E, D, B
   C) E, A, B
   D) E, A, C, B

10. The critical path algorithm is
    A) an approximate and inefficient algorithm.
    B) an optimal and efficient algorithm.
    C) an approximate and efficient algorithm.
    D) an optimal and inefficient algorithm.

11. Let w be the minimum weight among all edge weights in an undirected connected graph. Let e be a specific edge of weight w. Which of the following is FALSE?
    A) There is a minimum spanning tree containing e.
    B) If e is not in a minimum spanning tree T then in the cycle formed by adding e to T, all edges have the same weight.
    C) Every minimum spanning tree has an edge of weight w.
    D) e is present in every minimum spanning tree.

12. A minimal spanning tree of a graph G is.... ?
    A) A spanning sub graph
    B) A tree
    C) Minimum weights
    D) All of above

13. A tree having a main node, which has no predecessor is.... ?
    A) Spanning tree
    B) Rooted tree
    C) Weighted tree
    D) None of these

14. If a graph is a tree then
    A) it has 2 spanning trees
    B) it has only 1 spanning tree
    C) it has 4 spanning trees
    D) it has 5 spanning trees

15. Any two spanning trees for a graph
    A) Does not contain same number of edges
    B) Have the same degree of corresponding edges
    C) contain same number of edges
    D) May or may not contain same number of edges

16. A sub graph of a graph G that contains every vertex of G and is a tree is called
    A) Trivial tree
    B) empty tree
    C) Spanning tree
    D) Minimum Spanning tree

17. The minimum number of spanning trees in a connected graph with “n” nodes is.....
    A) n-1
    B) n/2
    C) 2
    D) 1

18. A minimal spanning tree of a graph G is
    A) A spanning sub graph
    B) A tree
    C) Minimum weights
    D) All of above